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Araştırma Makalesi / Research Article

Volatility Forecasting in Borsa Istanbul: A Comparative Analysis of Participation and Conventional Indices*

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Abstract

Volatility is a key input for investment decisions and portfolio management. Volatility modeling and forecasting are essential for risk analysis and risk management in financial markets. In this study, we investigate volatility characteristics of the four indices in Borsa Istanbul (Istanbul Stock Exchange) using GARCH family models. The primary motivation of this research is to model and forecast the volatility of participation and conventional indices. First, we estimate the model parameters using 809 days of data and explain the volatility features of the index return series. Second, we generate 57 days of forecast series and calculate realized volatility in the out-of-sample period. Finally, we analyze the predictive power of the forecast series of the models in relation to realized volatility, calculated using intraday returns. We use measures such as RMSE (Root Mean Squared Error), QLIKE (Quasi-Likelihood Loss), and R-squared (R^2) to evaluate the predictability of GARCH models. The results reveal that participation indices exhibit distinct volatility characteristics and that asymmetric models—especially EGARCH and TGARCH—produce satisfactory predictions.

Keywords: *Volatility Forecasting, GARCH Models, Participation Index, Borsa Istanbul.*

Borsa İstanbul'da Volatilite Öngörümlemesi: Katılım ve Konvansiyonel Endekslerin Karşılaştırmalı Analizi

Öz

Volatilite yatırım kararları ve portföy yönetimi için temel girdilerden biridir. Volatilite modellemesi ve öngörümlemesi finansal piyasalarda risk analizi ve yönetimi açısından oldukça önemlidir. Bu çalışmada, Borsa İstanbul'daki dört farklı endeksin volatilite özelikleri GARCH tipi modellerle incelenmiştir. Araştırmanın temel motivasyonu katılım ve konvansiyonel endekslerin volatilitesini modellemek ve ilgili endekslerde öngörümleme yapmaktır. Öncelikle 809 günlük getiri verisiyle GARCH modellerinin parametreleri tahmin edilmiş ve endeks getiri serilerinin volatilite özelikleri incelenmiştir. Daha sonra örneklem dışı 57 günlük tahmin serileri elde edilmiş ve günlük bazda gerçekleşmiş volatilite hesaplanmıştır. Son olarak modellerin tahmin serilerinin gün içi getirilerle hesaplanan gerçekleşmiş volatiliteyi öngörümleme gücü analiz edilmiştir. GARCH modellerinin öngörümleme gücünü incelemek üzere RMSE (Root Mean Squared Error), QLIKE (Quasi-Likelihood Loss) ve R-squared (R^2) ölçütleri hesaplanmıştır. Sonuçlar katılım endekslerinin, özelikle Katılım 30 endeksinin geleneksel endekslere kıyasla volatilite karakteristiğinin farklı olduğunu göstermektedir. Ayrıca EGARCH ve TGARCH asimetrik modellerin kullanılan diğer modellere göre gerçekleşen volatiliteyi öngörümleme açısından daha başarılı tahminler ürettiği izlenmektedir.

Anahtar Kelimeler: *Volatilite Tahmini, GARCH Modelleri, Katılım Endeksi, Borsa İstanbul.*

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INTRODUCTION

Volatility refers to unexpected or unpredictable fluctuations in the prices of financial assets. Typically, in financial markets, volatility is concerned with the spread of asset returns (Kayalıdere, 2013; Poon, 2005). Volatility is a metric closely monitored by financial market participants. As input, it plays a vital role in investment decisions and portfolio management. In this regard, volatility estimation is an attractive field of study for researchers. The primary motivation for study in this field is to model and forecast volatility. GARCH (Generalized Autoregressive Conditional Heteroskedasticity) family models are widely used for forecasting volatility. Empirical studies reveal that GARCH models successfully capture dynamic changes in a series, can model unexpected shocks to the series, accurately reflect the asymmetry and leverage effect in volatility, and have high forecasting capabilities (Pilbeam & Langeland, 2015).

In this study, we model and forecast the volatility of the Borsa Istanbul Participation 30 (XK030), Borsa Istanbul Participation 50 (XK050), Borsa Istanbul 30 (XU030), and Borsa Istanbul 100 (XU100) indices using symmetric and asymmetric GARCH models. We used GARCH (Bollerslev, 1986), Threshold GARCH (TGARCH) (Zakoian, 1994), Exponential GARCH (EGARCH) (Nelson, 1991), Asymmetric Power ARCH (APARCH) (Ding et al., 1993), and EGARCH-M (EGARCH in Mean) (Engle et al., 1987) models for volatility modeling and forecasting. Additionally, we calculated Root Mean Squared Error (MSE), Quasi-Likelihood Loss (QLIKE) and R-square (R^2) values for forecasting performance of the models in consistent with Xiao et al. (2018) methodology. XK030 and XK050 are called Islamic index generally, and they serve as an alternative for financial markets and are of interest to different segments in the financial system These indices are composed by Borsa Istanbul according to qualitative and quantitative screening process (Yıldız, 2020) (See Appendix 1). XU030 and XU100 are conventional indices, and XU100 is referred to as a market index.

The primary motivation of this research is to estimate volatility in participation and conventional indices using GARCH models and to examine the forecasting performance of these models in terms of volatility. The secondary objective is to analyze the volatility features of the indices based on the estimated parameters. The study is structured around three research questions: (1) Do participation and conventional indices in Borsa Istanbul exhibit different volatility characteristics? (2) Which volatility forecasting model provides the most accurate results, and does its performance vary depending on the indices? (3) Is the volatility of participation indices more predictable and consistent over time than conventional indices? The findings of this study will address the research questions.

Several empirical studies in the literature examine the modeling and forecasting of volatility in financial time series. Kashyap (2023) states that researchers commonly use GARCH-type models for volatility modeling and forecasting, and they compared the forecasting performance using statistical measures. Examples of these studies include Kayalidere (2013), Sharma and Vipul (2015), Nilsson (2017), Herrera et al. (2018), Hattori (2020), Liu et al. (2021), Lyocsa et al. (2021), and Chai et al. (2023). Kayalidere (2013) conducted a study forecasting volatility using MA (Moving Average), EWMA (Exponential Weighted Moving Average), and GARCH models in three emerging markets. The author emphasized that asymmetric models accurately forecast realized volatility calculated using intraday data. A similar study was conducted by Sharma and Vipul (2015) on 21 indices, which, contrary to Kayalidere (2013), concluded that the standard GARCH model produces better forecasting results than asymmetric models. Nilsson (2017) compared the EWMA and GARCH models and reported that the best model for volatility estimates is APARCH on the Swedish Stock Market. Herrera et al. (2018) used the RiskMetrics framework and GARCH models to estimate and

forecast the volatility of crude oil return series. The researchers tested the out-of-sample performance of the models using statistical measures, such as MSE and QLIKE, for short, medium, and long periods. The results of the study indicated that the EGARCH is the best forecasting model for the medium term, and the student's t distribution was optimal. Hattori (2020) applied GARCH models to Bitcoin return series. The author estimated the parameters using daily returns and calculated realized volatility using intraday 5-minute returns. The results of Hattori (2020)'s study emphasize the superiority of the EGARCH and APARCH models, namely asymmetric models, for forecasting volatility. Liu et al. (2021) reported that high-frequency data were more useful in volatility modeling and improved out-of-sample forecasting performance. Lyocsa et al. (2021) provide empirical evidence to the contrary, arguing that volatility forecasting with high-frequency data outperforms in short periods. Chai et al. (2023) proposed a semiparametric volatility model, emphasizing that forecast accuracy can be enhanced with high-frequency data.

We consider it important to review empirical studies on Islamic indices. Ahmed et al. (2016) estimated the volatility of Islamic and conventional indices selected from the Karachi Stock Exchange's (KSE) and Dow Jones indices. The results showed that the APARCH is the optimal model in conventional indices, while the EGARCH and standard GARCH are the most suitable models in Islamic indices for volatility estimation. Abdalla and Winker (2012) conducted parameter estimation using GARCH-type models and analyzed the volatility of the index series in the Sudan (KSE) and Egypt (CASE) stock markets. Using the daily price data of the indices, the researchers examined the main features, effects, and the risk-return trade-off.

Many researchers have reported their estimation and forecasting results obtained from GARCH models with different parameters and hybrid models trained with neural networks. More recently, machine learning-based models have been using volatility forecasting. Examples include Bildirici and Ersin (2009), Ou and Wang (2010), Hajizadeh et al. (2011), Kristjanpoller and Minutolo (2015), Katsiampa (2017), Rahimikia and Poon (2020), Christensen et al. (2023), and Zhang et al. (2024).

This study involved volatility modeling using daily returns data, and we estimated the parameters of GARCH models. Additionally, we evaluated the volatility forecasting performance of the models using the realized volatility proxy. We use intraday data for calculating realized volatility, as this aligns with the relevant empirical literature.

1. DATA, METHODOLOGY, AND FINDINGS

1.1. Data and Methods

Four indices were included in the scope of this research. The 809-day logarithmic returns of the indices, as of January 4, 2021, and March 29, 2024, were used to estimate the model parameters. Logarithmic returns (r_t) were calculated from closing price data (Equation 1).

$$r_t = ln\left(\frac{p_t}{p_{t-1}}\right) \quad (1)$$

where p_t donates the closing price at time t. ln represents natural logarithm function.

For the forecasting period, returns were calculated using intraday 5-minute price data from April 1, 2024, to June 28, 2024. There were 5577 intraday observations. The square of intraday returns is used as a measure of volatility (σ_t^2), which is consistent with Andersen and Bollerslev (1988). (Equation 2).

$$\sigma_t^2 = \sum_{i=1}^{n} r_i^2 \quad (2)$$

where r denotes the intraday returns.

After the calculation, a 57-day realized volatility series was obtained. Thus, the forecasting performance of the models for realized volatility is analyzed by solving regression equations. 20 equations were solved for the application (Equation 3). All data was collected from the Refinitiv Eikon DataStream. The computations were conducted using EViews 11 and MS Excel.

$$\sigma_t^2 = \vartheta + \tau\hat{\sigma}_t^2 + \varepsilon_t \quad (3)$$

where σ_t^2 denotes the realized volatility series. $\hat{\sigma}_t^2$ represents forecasting series.

For modeling and forecasting volatility, we used GARCH, TGARCH, EGARCH, APARCH, and EGARCH-M models in this research. The general specification for the conditional variance equations is as follows (Bolgün & Akçay, 2009; Poon, 2005):

GARCH (1,1): $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2$ (4)

where ω donates the constant term. α (reaction parameter) represents the effect of unexpected change in return on volatility, β (continuous parameter) represents how the change in the previous volatility affects the volatility in the following period. $\alpha + \beta$ denotes the persistence of volatility.

TGARCH (1,1): $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma N_{t-1} u_{t-1}^2 + \beta\sigma_{t-1}^2$ (5)

where ω, α, and β denote the constant term, ARCH and GARCH parameter, respectively. If $u_{t-1}^2 < 0$, N_{t-1} is equal to 1. If $u_{t-1}^2 \geq 0$, N_{t-1} is equal to 0. $\gamma > 0$ donates that negative shocks increase volatility more than posivite shocks.

EGARCH (1,1): $log(\sigma_t^2) = \omega + \beta log(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$ (6)

where ω, α, and β denote the constant term, ARCH and GARCH parameter, respectively. $\gamma \neq 0$ represents that positive and negative shocks have different effects on volatility. If γ is negative, negative shocks increase volatility more than positive shocks.

APARCH (1,1): $\sigma_t^\delta = \omega + \alpha(|u_{t-1}| - \gamma u_{t-1})^\delta + \beta\sigma_{t-1}^\delta$ (7)

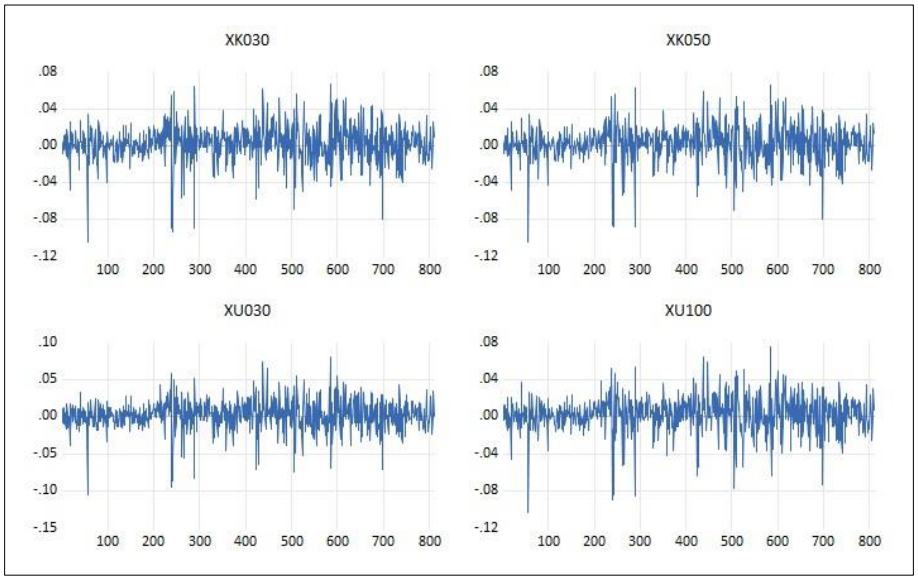
where ω, α, and β denote the constant term, ARCH and GARCH parameter, respectively. If $\gamma > 0$, negative shocks increase volatility more than positive shocks. This model includes the asymmetry parameter (γ) and power parameter (δ). The power parameter makes it possible to model different volatility levels.

The descriptive statistics of the indices for the estimation period are presented in Table 1. According to the statistical features of the indices, the mean returns are positive, exhibit a left-skewed distribution, are leptokurtic, and do not follow a normal distribution, as expected. The return distribution graphs of the indices are shown in Graph 1.

Table 1: Descriptive Statistics

Estimation Period (01/04/2021–03/29/2024)				
	XK030	**XK050**	**XU030**	**XU100**
Mean	0.002194	0.002180	0.002222	0.002254
Median	0.003048	0.002931	0.002164	0.002761
Maximum	0.066894	0.065889	0.080433	0.075718
Minimum	-0.104589	-0.104294	-0.104760	-0.103068
Std. Dev.	0.019284	0.018932	0.019705	0.018767
Skewness	-0.705099	-0.735594	-0.609272	-0.780083
Kurtosis	6.681969	6.718602	6.630241	6.963847
Jarque-Bera	524.0148	539.0769	494.2822	611.6782
Prob.	0.0000	0.0000	0.0000	0.0000
Observation	809	809	809	809

Figure 1: Return Distributions of Indices



Source: Created by Authors

The steps for the estimation period are as follows: The stationary test (ADF) was applied to logarithmic returns (Table 2). Suitable autoregressive (AR) and moving average (MA) mean equations are determined to ensure the adequacy and reliability of the volatility estimation (Table 3). The autocorrelation and ARCH effects in the residuals of the mean equation were tested for different lag levels. Accordingly, autoregressive conditional variance modeling was performed using the GARCH-type models. As observed in the Augmented Dickey-Fuller (ADF) unit root test results in Table 2, the index return series meets the stationarity requirement for modeling volatility. Test findings indicate that the return series are stationary and do not exhibit a unit root.

Table 2: Unit Root Test Results

Index	t-Statistic	Prob.
XK030	-27.98382	0.0000*
XK050	-28.09791	0.0000*
XU030	-28.93461	0.0000*
XU100	-28.74098	0.0000*

Notes: Test critical values are -3.438218 for 1%, -2.864902 for 5%, and -2.568615 for 10%. *Probability values indicate the rejection of the null hypothesis at the 1% significance level.

The logarithmic returns of the four indices were tested up to two lags, and the mean equation, including suitable AR and MA terms, was selected according to log likelihood, Akaike Information Criteria (AIC), and R-squared (Table 3). The mean equation of the indices was determined as ARMA (1,1).

Table 3: Statistics of Mean Equations

Index	ARMA(p,q)*	AIC Value	Log Likelihood	R-Squared
XK030	ARMA(1,1)	-5.052169	2047.602	Positive
XK050	ARMA(1,1)	-5.089444	2062.680	Positive
XU030	ARMA(1,1)	-5.011765	2031.259	Positive
XU100	ARMA(1,1)	-5.108758	2070.493	Positive

Note: *The coefficients of the mean equation are statistically significant.

We examined the autocorrelation problem in the variance of the residuals in the mean equation, also known as autoregressive conditional heteroskedasticity, using the ARCH-LM test for three, five, and ten lags. The results are shown in Table 4.

Table 4: Examination of ARCH Effect in Mean Equations

	XK030	XK050	XU030	XU100
Number of Lags: 3				
F-statistic	19.93141	19.18010	13.62166	13.00045
Prob. F (3,802)	0.0000*	0.0000*	0.0000*	0.0000*
Number of Lags: 5				
F-statistic	12.06419	11.60662	8.256661	7.805079
Prob. F(5,798)	0.0000*	0.0000*	0.0000*	0.0000*
Number of Lags: 10				
F-statistic	6.060143	5.827750	4.635464	4.103966
Prob. F(10,788)	0.0000*	0.0000*	0.0000*	0.0000*

Note: * Probability values indicate the rejection of the null hypothesis at the 1% significance level.

We observed the presence of ARCH effects in the residuals of the selected mean equation for the indices, suggesting that ARCH-type modeling is appropriate.

1.2. Findings

Table 5 shows the parameter estimates of GARCH models for XK030. The log-likelihood and AIC values indicate that the optimal model is EGARCH-M.

Table 5: Parameter Estimates of GARCH Models for XK030

Models	ω	α	β	γ	δ	LL	AIC
GARCH(1,1)	7.07E-05 (0.0007)	0.300540 (0.0001)	0.539497 (0.0000)	-	-	2135.920	-5.269604
TGARCH(1,1)	5.71E-05 (0.0002)	0.098565 (0.0570)	0.599101 (0.0000)	0.400539 (0.0025)	-	2149.494	-5.300727
EGARCH(1,1)	-1.279423 (0.0001)	0.291892 (0.0000)	0.868132 (0.0000)	-0.191344 (0.0000)	-	2152.363	-5.307829
APARCH (1,1)	0.005009 (0.3382)	0.176945 (0.0000)	0.760953 (0.0000)	0.742679 (0.0000)	0.789775 (0.0014)	2155.098	-5.312125
EGARCH-M (1,1)*	-1.348436 (0.0000)	0.298781 (0.0000)	0.859783 (0.0000)	-0.171122 (0.0001)	-	2155.545	-5.313231

Notes: Estimation results are given by testing the student's t distribution for TGARCH, EGARCH, APARCH, and EGARCH-M. GARCH model estimates are based on the GED distribution. The choice of error term distribution relies on the maximum LL (log likelihood) and minimum AIC (Akaike Information Criterion), which are frequently employed in GARCH-type modeling. *Coefficient estimate of $\sqrt{\sigma_n^2}$ term in mean equation: 0.451298 (Prob. 0.0161)

GARCH (1,1) model parameters are statistically significant. The sum of α and β parameters is less than 1.00, indicating that volatility is predictable (0.84003). Approximately 54% of the change in volatility may be attributed to conditional variance in the series from the previous period, while about 30% may be due toe past volatility shocks or unexpected returns. The long-run mean variance is 0.04%, and volatility is 2.1%. The half-life volatility in the series was calculated as 3.97 days (Table 13). This is a significant statistic for short-term investors and the market. The TGARCH (1,1) model parameters are statistically significant at the 1% level, except for the α parameter. Additionally, the coefficient is both positive and statistically significant. It indicates negative shocks have a greater impact on volatility than positive shocks. In other words, the market tends to perceive bad news more strongly. It is observed that negative shocks elicited a reaction on volatility up to the sum of α and γ (0.4991). The EGARCH (1,1) model coefficients are statistically significant. The asymmetry term (γ) is negative, indicating that volatility is more affected by negative shocks. The negative parameter indicates a leverage effect for XK030. The findings are consistent with the TGARCH model. The APARCH (1,1) model's outputs are in line with those of other asymmetric models. The power parameter reflects the effect of volatility shocks to the series (δ=0.789775). In the EGARCH-M model, we examined the rationality of the risk-return trade-off. The conditional standard deviation of the return is defined as an independent variable in the conditional mean equation. The estimates reveal that the coefficient is positive and significant at the 5% level. The findings indicate a rational risk-return trade-off in the Participation 30 Index.

The ARCH-LM test results are presented in Table 6. According to the test results, the models successfully eliminated the heteroskedasticity problem in the residuals. The lag lengths were determined as three, five, and ten.

Table 6: ARCH-LM Test Results of GARCH Models for XK030

	GARCH(1,1)	TGARCH(1,1)	EGARCH(1,1)	APARCH(1,1)	EGARCH-M(1,1)
Number of Lags: 3					
F-statistic	0.226736	0.312942	0.835540	1.262706	0.195268
Prob. F(3,801)	0.8778*	0.8160*	0.4745*	0.2860*	0.8996*
Number of Lags: 5					
F-statistic	0.202857	0.271195	0.531053	0.786359	0.141321
Prob. F(5,797)	0.9613*	0.9289*	0.7529*	0.5596*	0.9825*
Number of Lags: 10					
F-statistic	0.393174	0.381809	0.444961	0.558837	0.285505
Prob. F(10,787)	0.9499*	0.9548*	0.9242*	0.8479*	0.9844*

Note: *The probability values indicate that the null hypothesis cannot be rejected.

Table 7 shows the parameter estimates of the GARCH models for XK050. Regarding the log-likelihood and AIC values, the optimal model is the APARCH model. The GARCH (1,1) model parameters are statistically significant. The sum of α and β parameters is less than 1.00, implying that volatility is predictable (0.891889). It is observed that approximately 56% of the change in volatility can be explained by the previous period's conditional variance series. In comparison, approximately 34% can be explained by past volatility shocks or unexpected returns. The long-run mean variance is 0.06%, and the volatility is 2.38%. The half-life volatility in the series was calculated as 6.05 days (Table 13). It can be seen that the persistence of volatility in the XK050 is higher than in the XK030.

Table 7: Parameter Estimates of GARCH Models for XK050

Models	ω	α	β	γ	δ	LL	AIC
GARCH(1,1)	6.10E-05 (0.0006)	0.335382 (0.0001)	0.556507 (0.0000)	-	-	2159.537	-5.328061
TGARCH(1,1)	5.43E-05 (0.0001)	0.098686 (0.0623)	0.594542 (0.0000)	0.420111 (0.0020)	-	2167.516	-5.345336
EGARCH(1,1)	-1.261914 (0.0001)	0.288581 (0.0000)	0.870960 (0.0000)	-0.196399 (0.0000)	-	2170.767	-5.353385
APARCH (1,1)	0.008469 (0.2930)	0.163405 (0.0000)	0.781632 (0.0000)	0.813426 (0.0000)	0.634017 (0.0041)	2175.118	-5.361678
EGARCH-M (1,1)*	-1.345008 (0.0000)	0.294598 (0.0000)	0.860975 (0.0001)	-0.177469 (0.0001)		2173.945	-5.358775

Notes: Estimation results are presented by testing the student's t-distribution for all models. The choice of error term distribution relies on the maximum LL (log likelihood) and minimum AIC (Akaike Information Criterion), which are frequently employed in GARCH-type modeling. *Coefficient estimate of $\sqrt{\sigma_n^2}$ term in mean equation: 0.443111 (Prob. 0.0195)

The TGARCH (1,1) parameter estimates are statistically significant at the 1% level, except for the α parameter. Furthermore, γ indicates the presence of an asymmetric effect. Negative shocks have a greater impact on volatility than positive shocks. Negative shocks caused a reaction in volatility equal to the sum of α and γ (0.5187). The EGARCH (1,1) model parameters are statistically significant. The asymmetry term (γ) is negative, indicating that volatility is more affected by negative shocks. The asymmetric effect is important for risk management. Furthermore, we observe that the leverage effect exists for XK050, as indicated by the negative γ parameter. The findings are consistent with the TGARCH model. The APARCH (1,1) model outputs are statistically significant, except for γ parameter. The asymmetry term is positive, and there is a leverage effect in XK050 according to parameter estimates (γ=0.813426). In the EGARCH-M model, we observed the rationality of the risk-return trade-off. The estimates reveal that the coefficient is positive and significant at the 5% level. The findings indicate a rational risk-return trade-off in the Participation 50 Index.

Table 8: ARCH-LM Test Results of GARCH Models for XK050

	GARCH(1,1)	TGARCH(1,1)	EGARCH(1,1)	APARCH(1,1)	EGARCH-M(1,1)
Number of Lags: 3					
F-statistic	0.216174	0.378412	0.800651	1.315613	0.169370
Prob. F(3,801)	0.6421*	0.7686*	0.4937*	0.2681*	0.9171*
Number of Lags: 5					
F-statistic	0.271204	0.315031	0.516689	0.803919	0.122139
Prob. F(5,797)	0.9289*	0.9041*	0.7638*	0.5470*	0.9875*
Number of Lags: 10					
F-statistic	0.442369	0.392375	0.442828	0.558972	0.269921
Prob. F(10,787)	0.9257*	0.9503*	0.9254*	0.8478*	0.9875*

Note: *The probability values indicate that the null hypothesis cannot be rejected.

The ARCH-LM test results are presented in Table 8. According to test results, the models eliminated the heteroskedasticity problem in residuals. The lag lengths were determined as three, five, and ten. Table 9 shows the parameter estimates of GARCH models for XU030. Regarding the log likelihood and AIC values, the optimal model is APARCH.

Table 9: Parameter Estimates of GARCH Models for XU030

Models	ω	α	β	γ	δ	LL	AIC
GARCH(1,1)	1.28E-05 (0.0222)	0.069974 (0.0007)	0.899318 (0.0000)	-	-	2103.871	-5.190276
TGARCH(1,1)	6.30E-05 (0.0026)	0.085492 (0.0310)	0.659162 (0.0000)	0.218404 (0.0171)	-	2107.405	-5.196546
EGARCH(1,1)	-1.360030 (0.0018)	0.288215 (0.0000)	0.855610 (0.0000)	-0.118841 (0.0024)	-	2109.353	-5.201370
APARCH (1,1)	0.004828 (0.4402)	0.165957 (0.0000)	0.746112 (0.0000)	0.474423 (0.0004)	0.842251 (0.0063)	2110.410	-5.201510
EGARCH-M (1,1)*	-1.415896 (0.0011)	0.304794 (0.0000)	0.851032 (0.0000)	-0.075010 (0.0235)	-	2103.392	-5.184139

Notes: Estimation results are given by testing the student's t distribution for GARCH, TGARCH, EGARCH, and APARCH. EGARCH-M model estimates are based on the GED distribution. The choice of error term distribution relies on the maximum LL (log likelihood) and minimum AIC (Akaike Information Criterion), which are frequently employed in GARCH-type modeling. *Coefficient estimate of $\sqrt{\sigma_n^2}$ term in mean equation: 0.199447 (Prob. 0.1757)

In the GARCH (1,1) model, α and β parameter estimates are significant at the 1% level. The sum of α and β parameters is less than 1.00, indicating that volatility is predictable (0.969292). The persistence of volatility is higher than that of the other indices. It is observed that approximately 90% of the change in volatility can be explained by the previous period's conditional variance of the series, while approximately 7% can be explained by past volatility shocks or unexpected returns. The long-run mean variance is 0.04%, and the volatility is 2.04%. The half-life volatility in the series was calculated as 22.2 days (Table 13). We think that half-life volatility is a remarkable finding for the Borsa Istanbul 30 Index. The term asymmetry indicates the presence of an asymmetric effect ($\gamma > 0$), and EGARCH model outputs support this evidence. The EGARCH (1,1) model coefficients are statistically significant. Negative shocks caused a reaction in volatility up to α-γ, namely up to 0.4070506. Additionally, this is negative; it means that volatility is more affected by negative shocks. It implies that volatile movements are largely downside. We observed the leverage effect in XK030. The APARCH (1,1) model outputs are consistent with the EGARCH model estimates. The power parameter (δ) is higher than other indices, reflecting the effect of volatility shocks to series. EGARCH-M (1,1) model estimates show that the coefficient is positive but not statistically significant. This implies that the risk-return trade-off is theoretically rational.

The ARCH-LM test results are presented in Table 10. According to the test results, the models effectively eliminate the heteroskedasticity problem in the residuals. The lag lengths were determined as three, five, and ten.

Table 10: ARCH-LM Test Results of GARCH Models for XU030

	GARCH(1,1)	TGARCH(1,1)	EGARCH(1,1)	APARCH(1,1)	EGARCH-M(1,1)
Number of Lags: 3					
F-statistic	0.420367	0.166244	0.487666	1.005045	0.113053
Prob. F(3,801)	0.7384*	0.9191*	0.6909*	0.3899*	0.9525*
Number of Lags: 5					
F-statistic	0.312168	0.106104	0.303832	0.630168	0.089559
Prob. F(5,797)	0.9058*	0.9909*	0.9107*	0.6768*	0.9939*
Number of Lags: 10					
F-statistic	0.391616	0.394668	0.419056	0.571546	0.392632
Prob. F(10,787)	0.9506*	0.9493*	0.9378*	0.8379*	0.9502*

Note: *The probability values indicate that the null hypothesis cannot be rejected.

Table 11 shows the parameter estimates of GARCH models for XU100. Regarding log likelihood and AIC values, the optimal model is APARCH. The GARCH (1,1) model parameters are statistically significant. The sum of α and β parameters is less than 1.00, meaning that volatility is predictable (0.871929). It can be stated that approximately 68% of the change in volatility may cause conditional variance of the series in the previous period, while approximately 19% may cause past volatility shocks or unexpected returns. The long-run mean variance is 0.04%, and the volatility is 2.03%. The half-life volatility in the series was calculated as 5.05 days (Table 13).

Table 11: Parameter Estimates of GARCH Models for XU100

Models	ω	α	β	γ	δ	LL	AIC
GARCH(1,1)	5.27E-05 (0.0067)	0.194739 (0.0008)	0.677190 (0.0000)	-	-	2152.243	-5.310008
TGARCH(1,1)	6.72E-05 (0.0003)	0.046701 (0.2634)	0.609046 (0.0000)	0.403013 (0.0038)	-	2164.396	-5.337615
EGARCH(1,1)	-1.662255 (0.0007)	0.302957 (0.0001)	0.820777 (0.0000)	-0.142719 (0.0014)	-	2159.400	-5.325249
APARCH (1,1)	0.006350 (0.3067)	0.146184 (0.0000)	0.788055 (0.0000)	0.951958 (0.0000)	0.733301 (0.0012)	2174.029	-5.358982
EGARCH-M (1,1)*	-1.786857 (0.0002)	0.299216 (0.0003)	0.806561 (0.0000)	-0.150754 (0.0003)	-	2160.491	-5.325473

Notes: Estimation results are given by testing the student's t distribution for GARCH, TGARCH, EGARCH, and APARCH. EGARCH-M model estimates are based on the GED distribution. The choice of error term distribution relies on the maximum LL (log likelihood) and minimum AIC (Akaike Information Criterion), which are frequently employed in GARCH-type modeling. *Coefficient estimate of $\sqrt{\sigma_n^2}$ term in mean equation: 0.324308 (Prob. 0.0981)

The TGARCH (1,1) model coefficients are at a 1% significance level except for the α parameter. Negative shocks have a greater impact on volatility than positive shocks ($\gamma > 0$). The XU100 has a negative asymmetry term. A negative asymmetry term (γ) indicates that there is a leverage effect in the XU100. The EGARCH (1,1) model outputs are consistent with the TGARCH model. The APARCH (1,1) parameter estimates are statistically significant except for ω. Model outputs are consistent with other asymmetric model estimates. The power parameter shows the effect of volatility shocks to the series (δ=0.733301). In the EGARCH-M (1,1) model, the rationality of the risk-return trade-off for XU100 was examined. The estimates show that the coefficient is positive but not statistically significant. This implies that the risk-return trade-off is theoretically rational. The outputs of the EGARCH-M model support rational pricing in all indices.

Table 12 provides the ARCH-LM test results. Accordingly, the models eliminate the heteroskedasticity problem in the residuals. The lag lengths were determined as three, five, and ten. Additionally, Ljung-Box test statistics are presented in Appendix 2.

Table 12: ARCH-LM Test Results of GARCH Models for XU100

	GARCH(1,1)	TGARCH(1,1)	EGARCH(1,1)	APARCH(1,1)	EGARCH-M(1,1)
Number of Lags: 3					
F-statistic	0.099473	0.104594	0.297271	1.134312	0.089564
Prob. F(3,801)	0.9603*	0.9574*	0.8274*	0.3343*	0.9658*
Number of Lags: 5					
F-statistic	0.158402	0.130747	0.200257	0.719240	0.091484
Prob. F(5,794)	0.9775*	0.9854*	0.9624*	0.6091*	0.9936*
Number of Lags: 10					
F-statistic	0.388630	0.274732	0.269640	0.483511	0.239014
Prob. F(10,784)	0.9519*	0.9866*	0.9875*	0.9013*	0.9923*

Note: *The probability values indicate that the null hypothesis cannot be rejected.

2. RESULTS

Table 13 shows a summary of the estimation results of GARCH models. The results indicate that the volatility persistence was highest in XU030 and lowest in XK030. The half-life volatility is 22.2 days for XU030, which is relatively high compared with the other indices. On the other hand, it is remarkable that the long-term average volatility of the index is 2.04%. An asymmetric effect was observed for all indices. In other words, negative market conditions (i.e., negative information and news) have a greater impact on volatility. Asymmetric models are the most effective volatility estimation models for all indices. The risk-return trade-off is statistically significant and rational in the participation indices. On the contrary, the risk-return trade-off in conventional indices is rational but statistically insignificant.

Table 13: Summary Outputs of GARCH Models

Indices	$\alpha + \beta^{*}$	HV**	Avg. volatility (%)***	Optimal Model****	Asymmetric Effect	Risk-Return Trade-off
XK030	0.840030	3.97	2.10	EGARCH-M	✓	✓
XK050	0.891889	6.05	2.38	APARCH	✓	✓
XU030	0.969292	22.2	2.04	APARCH	✓	✓t
XU100	0.871929	5.05	2.03	APARCH	✓	✓t

Notes: *It refers to the volatility persistence. It is calculated using GARCH (1,1) model parameters.

** It refers to the half-life volatility. It is calculated using $HV = log\,(2)/log\,(\frac{1}{\alpha+\beta})$ equation.

*** It refers to the long-term daily average volatility. It is calculated using $\overline{\sigma} = \sqrt{\omega/(\,1-\alpha-\beta)}$ equation.

****The best model was selected according to log likelihood and AIC values.

(t) The coefficient is statistically insignificant, but it is theoretically acceptable.

The forecasting performance of GARCH models depends on their accuracy in predicting realized volatility. In this study, we used different measures, RMSE (Root Mean Squared Error), QLIKE (Quasi-Likelihood Loss), and R-squared (R^2), to evaluate the predictability of GARCH models. Table 14 provides a computation of the results of loss functions and R^2 coefficients for the out-of-sample period. The RMSE and QLIKE results show that asymmetric volatility models, such as TGARCH and EGARCH, perform better in predicting participation indices than others. Additionally, we observed that the GARCH model yields acceptable predictions for conventional indices. In other words, the symmetric model prediction ability is higher than the EGARCH and APARCH models for conventional indices. In the regression equations, the estimated variance series is involved as the explanatory variable and the realized volatility as the dependent variable (see Equation 3). A high R^2 coefficient indicates the success of the models in accurately forecasting the realized volatility. The regression results demonstrate that the EGARCH model performs successfully in forecasting volatility across all indices. The R^2 coefficient indicates that the APARCH model is appropriate for indices. It can be stated that XK030 volatility is more predictable than the others.

Table 14: RMSE, QLIKE, and R^2 of Volatility Forecasts for Indices

Models	RMSE*	QLIKE**	R^2
		XK030	
GARCH-ged	2.10E-04	-8.59727	0.00094
TGARCH-t	1.64E-04	-8.62266	0.07764
EGARCH-t	1.67E-04	-8.60388	0.17952
APARCH-t	1.93E-04	-8.57542	0.15735
EGARCH-M-t	3.22E-04	-8.37397	0.11490
		XK050	
GARCH-t	2.29E-04	-8.62340	0.00070
TGARCH-t	1.62E-04	-8.67101	0.04448
EGARCH-t	1.65E-04	-8.65363	0.11640
APARCH-t	2.06E-04	-8.61003	0.10526
EGARCH-M-t	3.21E-04	-8.40854	0.06908
		XU030	
GARCH-t	1.52E-04	-8.48895	7.3E-05
TGARCH-t	1.46E-04	-8.49971	0.07408
EGARCH-t	1.55E-04	-8.48587	0.10529
APARCH-t	1.70E-04	-8.46718	0.07793
EGARCH-M-ged	1.55E-04	-8.48608	0.06804
		XU100	
GARCH-t	1.67E-04	-8.64104	0.00533
TGARCH-t	1.43E-04	-8.66134	0.09815
EGARCH-t	1.62E-04	-8.63578	0.13024
APARCH-t	1.71E-04	-8.62948	0.10012
EGARCH-M-ged	1.94E-04	-8.57704	0.10251

Note: $*RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{V}_t - FV_t)^2}$ where $\hat{V}_t, FV_t, T$ denote realized volatility, forecast volatility, number of periods respectively.

$**QLIKE = \frac{1}{T}\sum_{t=1}^{T}\left(log(FV_t) + \frac{\hat{V}_t}{FV_t}\right)$ where $\hat{V}_t, FV_t, T$ denote realized volatility, forecast volatility, number of periods respectively.

3. CONCLUSION

In this study, we conducted volatility modeling and examined the forecasting performance of GARCH family models using loss functions such as RMSE and QLIKE and regression analysis. The results reveal that participation and conventional indices exhibit different volatility structures, and the Participation 30 Index is more predictable than other indices. In this way, the Participation 30 Index can be considered a stable market based on its limited and homogeneous structure.

We can emphasize that asymmetric models, particularly EGARCH and TGARCH, yield accurate predictions. In other words, the models are still relevant for volatility forecasting. Some findings are consistent with Hattori's (2020) results. The author's empirical finding shows that asymmetric volatility models have more predictive power for Bitcoin data. The findings of our study support the model selection proposed by Hajizadeh et al. (2012) as the best.

It is known that the underperformance of conventional indices, especially during crisis periods, has increased the popularity of Islamic indices. Asutay et al. (2022) provide empirical evidence for this. In this respect, it is essential to highlight the volatility characteristics of participation and conventional indices.

It is necessary to highlight certain limitations of this study. First, volatility modeling is conducted with basic GARCH family models. Second, the entire period was modeled under a single regime. Therefore, structural breaks/changes were not explicitly integrated into the volatility estimation process. Third, this research specifically examines the four indices of Borsa Istanbul. Future research may focus on different participation or Islamic indices and compare their volatility characteristics with those of conventional indices by employing advanced approaches, such as stochastic volatility modeling. Additionally, researchers may also consider modeling and forecasting volatility under multiple regimes by employing Markov-Switching GARCH models.

AUTHOR STATEMENTS

Statement of Research and Publication Ethics

This study has been prepared in accordance with the principles of scientific research and publication ethics.

Ethics Committee Approval

This research does not require ethics committee approval.

Author Contributions

This article is based on the first author's doctoral dissertation, conducted under the supervision of the second author. The third author provided contributions to the research design and methodology.

Conflict of Interest

There is no conflict of interest for the authors or third parties arising from the study.

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Appendix 1 Steps of Screening Process for Participation Indices

1) Stocks that are not traded in the BIST Stars Market, BIST Main Market or Sub-Market at the start of the index period are excluded. In other words, the status of the market in which the companies are traded is taken into consideration.
2) Companies are excluded if they are ethically problematic, negatively affect people and other living beings, have a negative impact on their mental and physical health, and are intensive in money trading and interest-bearing transactions. The activities are as follows:

- Alcoholic beverages (both production and trade)
- Narcotic substances except for medical purposes (both production and trade)
- Gambling and gambling-related activities
- Pigs and their products (both production and trade)
- Financial transactions with interest
- Publication contrary to morality and Islamic values
- Entertainment, hotel management and similar activities that are not in accordance with Islamic values
- Activities that impose substantial harm on the environment and living beings
- Biological/genetic activities to change human nature
- Tobacco products that are harmful to health (production, wholesale trade and distribution)

3) Shares with certain privileges, such as profit or liquidation rights or usufruct shares, are excluded.
4) Companies that ignoring Islamic values, supporting acts that violate human rights and humanitarian values, in particular inherent right to life, are excluded. (Companies are also excluded if they are found guilty in these issues.)
5) Companies are excluded if they are considered directly contrary to the Participation Finance Principles are excluded.
6) In the screening of financial ratios, companies' links to sectors rated as completely inappropriate in their interest-bearing assets and liabilities are limited to certain reference values to ensure compliance with the index. Quantitative screening values as are follows:

- Revenues obtained from non-permissible activities / Total revenues < 5.00%
- Interest-bearing assets / higher of market value or total assets < 33.00%
- Interest-bearing debts / higher of market value or total assets < 33.00%

Source: Borsa Istanbul (2024)

Appendix 2 Ljung-Box Test Statistics

	GARCH(1,1)	TGARCH(1,1)	EGARCH(1,1)	APARCH(1,1)	EGARCH-M(1,1)
			XK030		
Number of Lags: 3					
Q-stat	0.6590	0.3849	0.5236	1.5376	2.2224
Prob.	0.417*	0.535*	0.469*	0.215*	0.136*
Number of Lags: 5					
Q-stat	2.6225	1.9775	1.4661	2.2402	2.7242
Prob.	0.454*	0.577*	0.690*	0.524*	0.436*
Number of Lags: 10					
Q-stat	4.2292	5.2935	5.4183	6.9815	8.6312
Prob.	0.836*	0.726*	0.712*	0.539*	0.374*
			XK050		
Number of Lags: 3					
Q-stat	0.4104	0.3455	0.6407	2.1142	2.8633
Prob.	0.522*	0.557*	0.423*	0.146*	0.091*
Number of Lags: 5					
Q-stat	1.5006	1.5969	1.3221	2.5018	3.1860
Prob.	0.682*	0.660*	0.724*	0.475*	0.364*
Number of Lags: 10					
Q-stat	3.1581	4.2938	4.6153	6.8686	8.3867
Prob.	0.924*	0.830*	0.798*	0.551*	0.397*
			XU030		
Number of Lags: 3					
Q-stat	1.3198	1.5702	1.9707	1.6914	2.3996
Prob.	0.251*	0.210*	0.160*	0.193*	0.121*
Number of Lags: 5					
Q-stat	6.2042	7.0218	6.8143	6.0213	7.9814
Prob.	0.102*	0.071*	0.078*	0.111*	0.046*
Number of Lags: 10					
Q-stat	6.5472	7.7449	7.4478	6.5872	8.2772
Prob.	0.586*	0.459*	0.489*	0.582*	0.407*
			XU100		
Number of Lags: 3					
Q-stat	2.1343	0.7931	2.2935	3.4652	1.7908
Prob.	0.144*	0.373*	0.130*	0.063*	0.181*
Number of Lags: 5					
Q-stat	7.4460	2.0776	3.3474	4.9802	3.0703
Prob.	0.059*	0.556*	0.341*	0.173*	0.381*
Number of Lags: 10					
Q-stat	7.7322	6.1233	7.6127	8.7775	6.8817
Prob.	0.460*	0.633*	0.472*	0.361*	0.549*

Note: *Probability values indicate the rejection of the null hypothesis.

Appendix 3 Realized Volatility of Indices

